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Effects of interface roughness and exchange splitting on shot noise in ferromagnet/superconductor junctions

Z C Dong[1,2], **D Y Xing**[1] **and Jinming Dong**[1]

[1] National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, China
[2] Department of Physics, Huaiyin Teachers College, Huaiyin 223001, China

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Abstract

The current fluctuations in ferromagnet/superconductor (FM/SC) junctions are studied for s- and d-wave pairing by taking into account the roughness of the interfacial barrier and exchange splitting in the FM. It is shown that the ferromagnetic exchange splitting gives rise to a decrease in the average current and the shot noise power; the noise power-to-current ratio is increased for $eV < \Delta_0$ but decreased for $eV > \Delta_0$ in FM/s-wave SC junctions (V being the bias voltage and Δ_0 the energy gap), while the ratio is increased rapidly with the exchange splitting at low voltages and tends towards the same value for high voltages in FM/d-wave SC junctions. The interface roughness is found to lead to a decrease in the average current and an increase in the noise power-to-current ratio.

1. Introduction

Electrical shot noise is the time-dependent fluctuation of the current around its mean value, due to the discreteness of charge carriers. Shot noise measurements can provide information on transport properties which cannot be obtained from usual resistance measurements. In the last few years, much attention has been paid to the study of shot noise in mesoscopic systems [1]. In particular, the shot noise in normal-metal/superconductor (NM/SC) junctions has been studied intensively [2–8]. It has been shown, through these works, that the Andreev reflection (AR) [9] and the charge transport by the Cooper pairs have a significant influence on the current fluctuation at low voltage. At the same time, the pairing symmetry of the SC plays an important role. For an NM/s-wave SC junction [2–6], the noise-to-current ratio is $4e$ at zero bias voltage and it recovers to the classical Schottky value of $2e$ for sufficiently higher voltage [10]. This feature can be understood from the fact that for energy smaller than the superconducting gap, an electron incident from the NM region is reflected as a hole at the NM/SC interface, resulting in the flow of a Cooper pair with charge $2e$ in the SC; while for energy much larger than the superconducting gap, the incident electron is transmitted as an electronlike quasiparticle with charge e in the SC. For an NM/d-wave SC junction with a nonzero angle between the a-axis of the crystal and the normal to the interface, the ratio is zero at zero voltage and $2e$ at a finite voltage [7, 8]. The anomalous behaviour in the NM/d-wave SC junctions is attributed to the formation of zero-energy bound states at the interface of the NM/d-wave SC junction.

Recently, transport properties in hybrid structures composed of ferromagnet (FM) and SC have received considerable theoretical [11–14] and experimental attention [15–18]; several important features have been revealed. The effects of the exchange interaction in the FM, the barrier height, and the Fermi wave-vector mismatch between FM and SC regions on differential conductances of FM/SC junctions have been investigated. Although there was a report on a study of the shot noise power in ballistic FM/s-wave SC junctions [11], the shot noise in general FM/SC junctions has not been systematically studied. It has not been clear how the ferromagnetic exchange splitting and interface roughness influence the shot noise. In this paper, we will address these important questions.

2. Shot noise in FM/SC junctions

Consider a two-dimensional FM/d-wave SC junction structure of semi-infinite FM and SC separated by a very thin insulating film at $x = 0$ as shown in figure 1. As in previous works [19, 20], we neglect for simplicity the self-consistency of the spatial distribution of the pair potential in the SC and take it as a step function [20]

$$\Delta(x) = \Delta_{\pm} = \Delta_0 \cos(2\theta_s \mp 2\alpha)\Theta(x)$$

where Δ_0 is a constant and $\Theta(x)$ is the Heaviside step function. θ_s is the angle between the interface normal and the momentum of the quasiparticle, as indicated in figure 1; α is the angle between the a-axis of the crystal and the interface normal, and Δ_+ (Δ_-) corresponds to the pair potentials for electronlike (holelike) quasiparticles. In the presence of interface roughness, the FM/SC interface barrier at $x = 0$ can be described by a δ-type potential $\delta(x)$ and a random roughness function $g(y)$ such that the barrier potential is given by $V(\boldsymbol{r}) = U\delta(x)g(y)$. In the Green's function approach under the 'white-noise' approximation, the self-energy contains an imaginary part independent of momentum. It then follows that the interface barrier may be

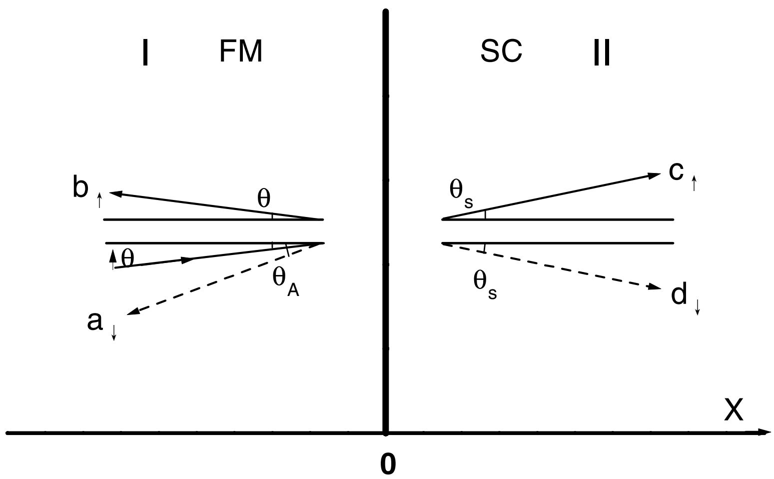

Figure 1. A schematic illustration of reflections and transmissions of quasiparticles in an FM/SC junction.

modelled by an effective interface potential [21]:

$$U\delta(x) = (U_0\hat{1} - \mathrm{i}P\hat{\tau}_3)\delta(x) \tag{1}$$

where $\hat{1}$ is the unit matrix and $\hat{\tau}_3$ a Pauli matrix. In this effective potential, U_0 indicates the barrier strength and P describes the scattering effect during tunnelling through the rough barrier.

We adopt the Bogoliubov–de Gennes (BdG) approach [22] to study the FM/SC junction. Within the Stoner model, the motion of conduction electrons inside the FM can be described by an effective single-particle Hamiltonian with an exchange energy h_0. In the absence of spin-flip scattering, the spin-dependent (four-component) BdG equations are decoupled into two sets of (two-component) equations: one for the spin-up electronlike and spin-down holelike quasiparticle wave functions $(u_\uparrow, v_\downarrow)$, the other for $(u_\downarrow, v_\uparrow)$. The BdG equation for $(u_\uparrow, v_\downarrow)$ is given by

$$\begin{bmatrix} H_0(\boldsymbol{r}) - h(\boldsymbol{r}) & \Delta(x,\theta) \\ \Delta^*(x,\theta) & -H_0^*(\boldsymbol{r}) - h(\boldsymbol{r}) \end{bmatrix} \begin{bmatrix} u_\uparrow(x,\theta) \\ v_\downarrow(x,\theta) \end{bmatrix} = E\left[\begin{pmatrix} u_\uparrow(x,\theta) \\ v_\downarrow(x,\theta) \end{pmatrix}\right]. \tag{2}$$

Here

$$H_0(\boldsymbol{r}) = -\hbar^2\nabla_{\boldsymbol{r}}^2/2m + V(\boldsymbol{r}) - E_F$$

with $V(\boldsymbol{r})$ the usual static potential, and $h(\boldsymbol{r}) = h_0\Theta(-x)$. The excitation energy E is measured relative to Fermi energy E_F.

Consider a beam of spin-up electrons incident on the interface at $x = 0$ from the FM at an angle θ to the interface normal. As shown in figure 1, there are four possible trajectories: normal reflection $(b_\uparrow)$, Andreev reflection $(a_\downarrow)$, transmission to SC as electronlike quasiparticles $(c_\uparrow)$, and transmission as holelike quasiparticles $(d_\downarrow)$. We wish to point out here that the AR coefficient $a_\downarrow$ is labelled with subscript $\downarrow$, because the AR results in an electron deficiency in the spin-down subband of the FM, even though it is at times called a spin-up hole. The general solutions of the BdG equation given by equation (2) are of the form

$$\Psi_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrm{e}^{\mathrm{i}q_\uparrow x\cos\theta} + a_\downarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathrm{e}^{\mathrm{i}q_\downarrow x\cos\theta_A} + b_\uparrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathrm{e}^{-\mathrm{i}q_\uparrow x\cos\theta} \tag{3a}$$

for $x < 0$, and

$$\Psi_{II} = c_\uparrow \begin{pmatrix} u_+\mathrm{e}^{\mathrm{i}\phi_+} \\ v_+ \end{pmatrix} \mathrm{e}^{\mathrm{i}k_F x\cos\theta_s} + d_\downarrow \begin{pmatrix} v_-\mathrm{e}^{\mathrm{i}\phi_-} \\ u_- \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_F x\cos\theta_s} \tag{3b}$$

for $x > 0$. Here

$$q_\uparrow \simeq \sqrt{2m(E_F + h_0)/\hbar^2}$$
$$q_\downarrow \simeq \sqrt{2m(E_F - h_0)/\hbar^2}$$

indicating different Fermi wave vectors for spin-up and spin-down subbands in the FM. In the SC region, the wave vectors of electronlike and holelike quasiparticles have been approximated by the Fermi wave vector:

$$k_F = \sqrt{2mE_F/\hbar^2}$$

and

$$(u_\pm)^2 = 1 - (v_\pm)^2 = (1 + \sqrt{1 - |\Delta_\pm/E|^2})/2$$
$$\phi_\pm = \cos^{-1}[\cos 2(\theta_s \mp \alpha)/|\cos 2(\theta_s \mp \alpha)|].$$

In the FM, $q_\uparrow$ is greater than $q_\downarrow$ due to the presence of the exchange splitting $2h_0$, neither of them being equal to k_F in the SC. However, they must satisfy the condition

$$q_\uparrow \sin\theta = q_\downarrow \sin\theta_A = k_F \sin\theta_s$$

to guarantee the conservation of the momentum components parallel to the interface. As a result, θ, θ_A, and θ_s differ from each other except when $\theta = 0$. In the present case, $q_\uparrow > k_F > q_\downarrow$, so $\theta_A < \theta_s < \theta$. With increasing θ, both θ_A and θ_s become large. As θ exceeds $\sin^{-1}(q_\downarrow/q_\uparrow)$, the x-component of the wave vector in the AR process, $\sqrt{(q_\downarrow^2 - q_\uparrow^2 \sin^2\theta)}$, will become purely imaginary, so the Andreev-reflected quasiparticles do not propagate; this is was referred to as virtual AR. Further, when $\theta > \sin^{-1}(k_F/q_\uparrow)$, the transmitted quasiparticles do not propagate and so net currents of the charge from FM to SC vanish.

All coefficients in equations (3*a*) and (3*b*) can be determined by imposing boundary conditions at $x = 0$:

$$\Psi_{II}(0) = \Psi_I(0)$$
$$(\mathrm{d}\Psi_{II}/\mathrm{d}x)_{x=0} - (\mathrm{d}\Psi_I/\mathrm{d}x)_{x=0} = 2mU\Psi_I(0)/\hbar^2.$$

We find

$$a_\downarrow = -4r_\uparrow u_- v_+ \mathrm{e}^{-\mathrm{i}\phi_+}/D \tag{4}$$

$$b_\uparrow = \{[2(\mathrm{i}z_1 + z_2) - r_\uparrow - 1][2(\mathrm{i}z_1 - z_2) - r_\downarrow + 1]v_+ v_- \mathrm{e}^{\mathrm{i}(\phi_- - \phi_+)} - [2(\mathrm{i}z_1 + z_2) - r_\uparrow + 1][2(\mathrm{i}z_1 - z_2) - r_\downarrow - 1]u_+ u_-\}/D \tag{5}$$

$$c_\uparrow = 2r_\uparrow[2(\mathrm{i}z_1 - z_2) - r_\downarrow - 1]u_- \mathrm{e}^{-\mathrm{i}\phi_+}/D \tag{6}$$

$$d_\downarrow = -2r_\uparrow[2(\mathrm{i}z_1 - z_2) - r_\downarrow + 1]v_+ \mathrm{e}^{-\mathrm{i}\phi_+}/D \tag{7}$$

with

$$D = [2(\mathrm{i}z_1 + z_2) + r_\uparrow + 1][2(\mathrm{i}z_1 - z_2) - r_\downarrow - 1]u_+ u_- - [2(\mathrm{i}z_1 + z_2) + r_\uparrow - 1][2(\mathrm{i}z_1 - z_2) - r_\downarrow + 1]v_+ v_- \mathrm{e}^{\mathrm{i}(\phi_- - \phi_+)}. \tag{8}$$

Here

$$r_\uparrow = q_\uparrow \cos\theta/(k_F \cos\theta_s)$$
$$r_\downarrow = q_\downarrow \cos\theta_A/(k_F \cos\theta_s)$$
$$z_1 = z_{10}/\cos\theta_s$$
$$z_2 = z_{20}/\cos\theta_s$$

with

$$z_{10} = mU_0/\hbar^2 k_F$$
$$z_{20} = mP/\hbar^2 k_F.$$

For spin-down electrons incident on the interface at $x = 0$, $a_\uparrow$, $b_\downarrow$, $c_\downarrow$, and $d_\uparrow$ can be similarly obtained, having expressions symmetric with equations (4)–(8). For an incident electron with spin down, since its wave vector is always smaller than that of the hole due to AR ($q_\downarrow < q_\uparrow$), θ is always greater than θ_A and so there is no virtual AR for all incident angles.

The shot noise power of the NM/SC junctions [5–7] is readily extended to the spin-dependent transport through an FM/SC junction. At zero temperature it is given by

$$P = \frac{1}{e}\int_0^{eV} \mathrm{d}E\, S_T(E) \tag{9}$$

where the differential shot noise includes an integral over the injection angle θ

$$S_T(E) = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \mathrm{d}\theta\, S(E)\cos\theta \tag{10}$$

for the d-wave SC, and

$$S_T(E) = 2\int_0^{\pi/2} \mathrm{d}\theta\ S(E)\cos\theta\sin\theta \tag{11}$$

for a three-dimensional s-wave SC, with

$$S(E) = \frac{4e^3}{h}\sum_{s=\uparrow,\downarrow} P_s\,[R_a(1-R_a) + (3R_a + R_c + R_d)(1 - R_a - R_c - R_d)]\,. \tag{12}$$

Here

$$P_\uparrow = (E_F + h_0)/2E_F$$
$$P_\downarrow = (E_F - h_0)/2E_F$$

are the polarization in the FM for the up and down spins, respectively;

$$R_a = |a_{\bar{s}}|^2 q_{\bar{s}} \cos\theta_A/(q_s\cos\theta)$$
$$R_b = |b_s|^2$$

are the AR and normal-reflection coefficients; and

$$R_c = (u_+^2 - v_+^2)|c_s|^2 k_F \cos\theta_s/(q_s\cos\theta)$$
$$R_d = (u_-^2 - v_-^2)|d_{\bar{s}}|^2 k_F \cos\theta_s/(q_s\cos\theta)$$

are the transmission coefficients of electronlike and holelike quasiparticles, with $\bar{s}$ indicating the spin opposite to s.

On the other hand, the average current is given by

$$I = \frac{1}{e}\int_0^{eV} \mathrm{d}E\ G_T(E) \tag{13}$$

where the differential conductance is [23]

$$G_T(E) = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \mathrm{d}\theta\ G(E)\cos\theta \tag{14}$$

for the d-wave SC, and

$$G_T(E) = 2\int_0^{\pi/2} \mathrm{d}\theta\ G(E)\cos\theta\sin\theta \tag{15}$$

for the s-wave SC, with

$$G(E) = \frac{2e^2}{h}\sum_{s=\uparrow,\downarrow} P_s(2R_a + R_c + R_d). \tag{16}$$

3. Calculated results and discussion

In what follows we calculate the average current I, the shot noise power P, and the noise power-to-current ratio P/I as functions of bias. Let us first study the effects of the exchange splitting on the shot noise in the absence of interface roughness by taking $z_{20} = 0$. For FM/s-wave SC junctions, it is found that with the exchange splitting increased, both the average current and the shot noise power are reduced, while the noise power-to-current ratio is increased for $eV < \Delta_0$ but decreased for $eV > \Delta_0$, as shown in figure 2. That the noise power-to-current ratio is smaller than $4e$ at zero bias stems from the fact that a smaller $z_{10} = 1$ was taken in figure 2. If z_{10} is taken to be a larger value, which corresponds to a higher potential barrier, this ratio will be closer to $4e$, as discussed below. For a FM/d-wave SC junction ($\alpha = \pi/4$),

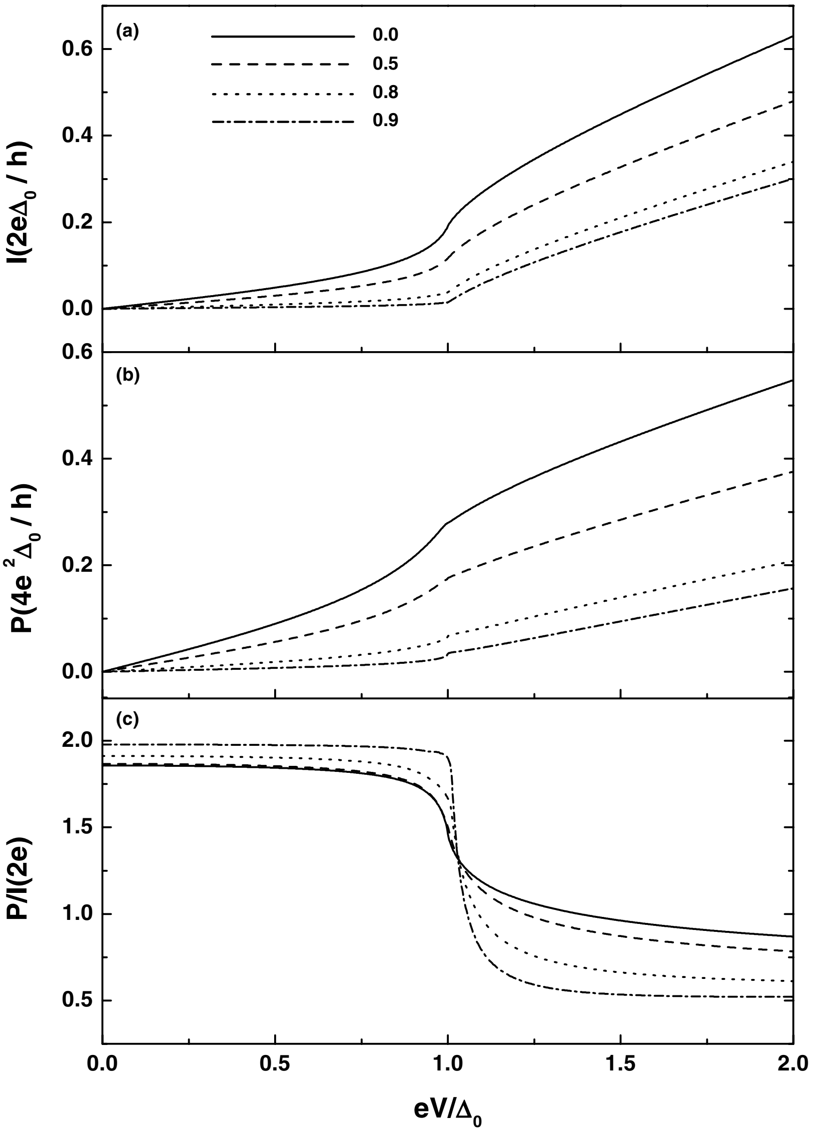

Figure 2. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of h_0/E_F in an FM/s-wave SC junction. Here $z_{10} = 1.0$ and $z_{20} = 0$.

as shown in figure 3, an increase in the exchange splitting gives rise to a decrease in I and P except close to zero bias voltage, but the ratio P/I is increased rapidly with the exchange splitting at low voltages and tends towards the same value for high voltages.

Next, we study the effects of the interface roughness on the shot noise. In the presence of the interface roughness, the scattering at the interface would lead to a decrease in R_a, R_c, and R_d. It follows from equation (16) that an increase in the interface roughness always gives rise to a decrease of I, regardless of the magnitude of z_{10}. As regards the shot noise power, however, the situation is more complicated, depending on the barrier strength. For large z_{10}, all

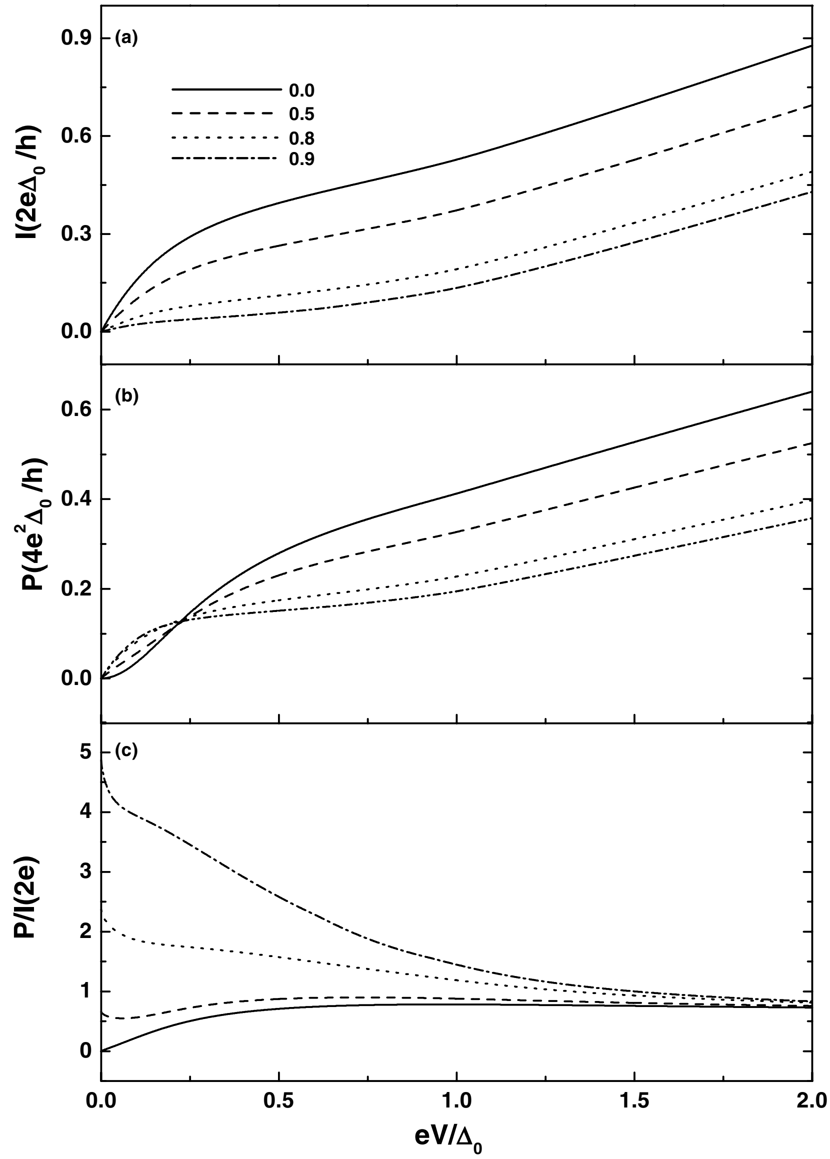

Figure 3. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of h_0/E_F in an FM/d-wave SC junction. Here $\alpha = \pi/4$, $z_{10} = 1.0$, and $z_{20} = 0$.

the coefficients R_a, R_c, and R_d are small, so in equation (12) the dominant factor determining the variation of $S(E)$ is the decrease of R_a and $(3R_a + R_c + R_d)$ with increasing z_{20}. In this case, as the interface roughness is increased, both I and P decrease. Due to the presence of the factors $(1 - R_a)$ and $(1 - R_a - R_c - R_d)$ in equation (12), the decrease rate of P is smaller than that of I, so P/I increases with z_{20}. This deduction has been confirmed by our numerical calculations. In reality, the variations in I and P due to the interface roughness are very small for high barriers, especially in the case of $z_{10} \gg z_{20}$. On the other hand, for very small z_{10} (e.g. $z_{10} = 0$), the normal-reflection coefficient is very small and $(R_a + R_c + R_d)$ is close to 1. For $eV < \Delta_0$, both R_c and R_d can be neglected and R_a is close to 1. In this case, the dominant factor in equation (12), which determines the variation of $S(E)$, is the increase of $(1 - R_a)$ with z_{20}. As a result, on increasing the interface roughness by increasing z_{20}, I decreases but P increases, so P/I is increased. Figures 4 and 5 show numerical results for I, P, and P/I as functions of bias for different z_{20} obtained by taking $z_{10} = 0$ and $h_0/E_F = 0.5$. It is found that the qualitative argument above is in good agreement with the numerical results.

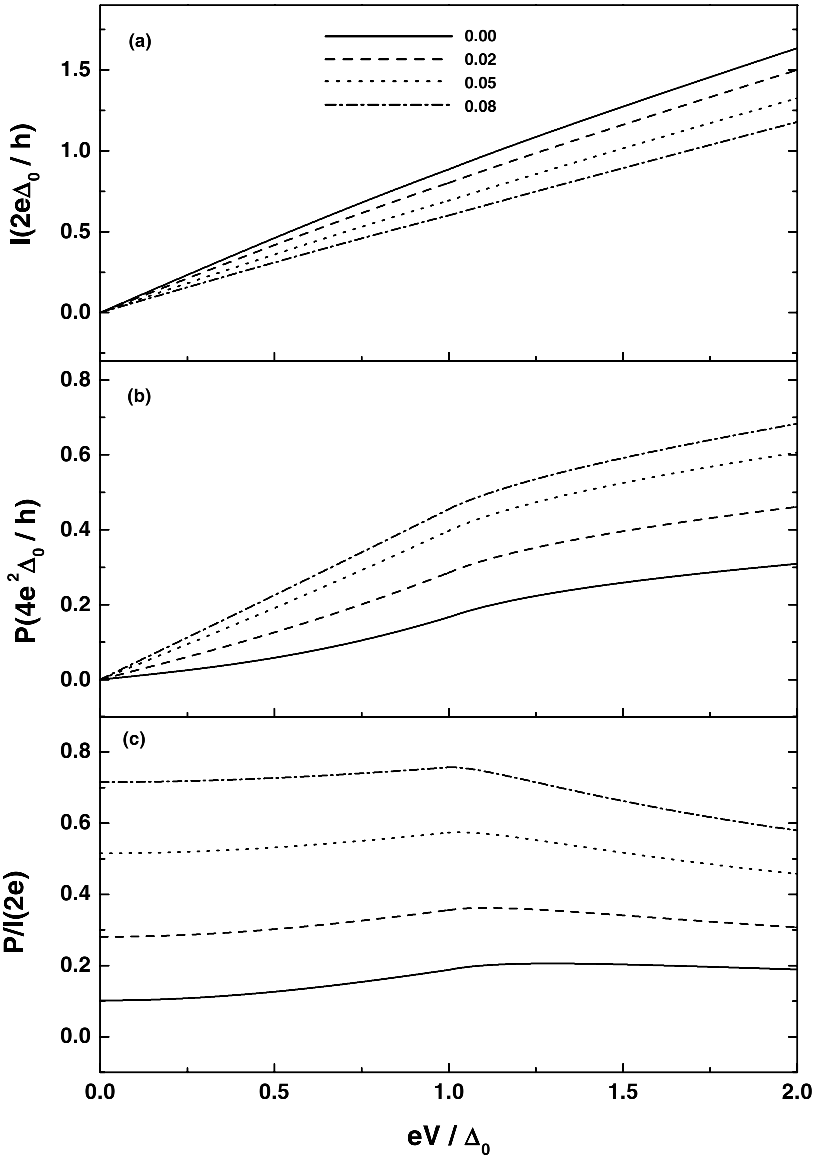

Figure 4. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of z_{20} in an FM/s-wave SC junction. Here $z_{10} = 0$ and $h_0/E_F = 0.5$.

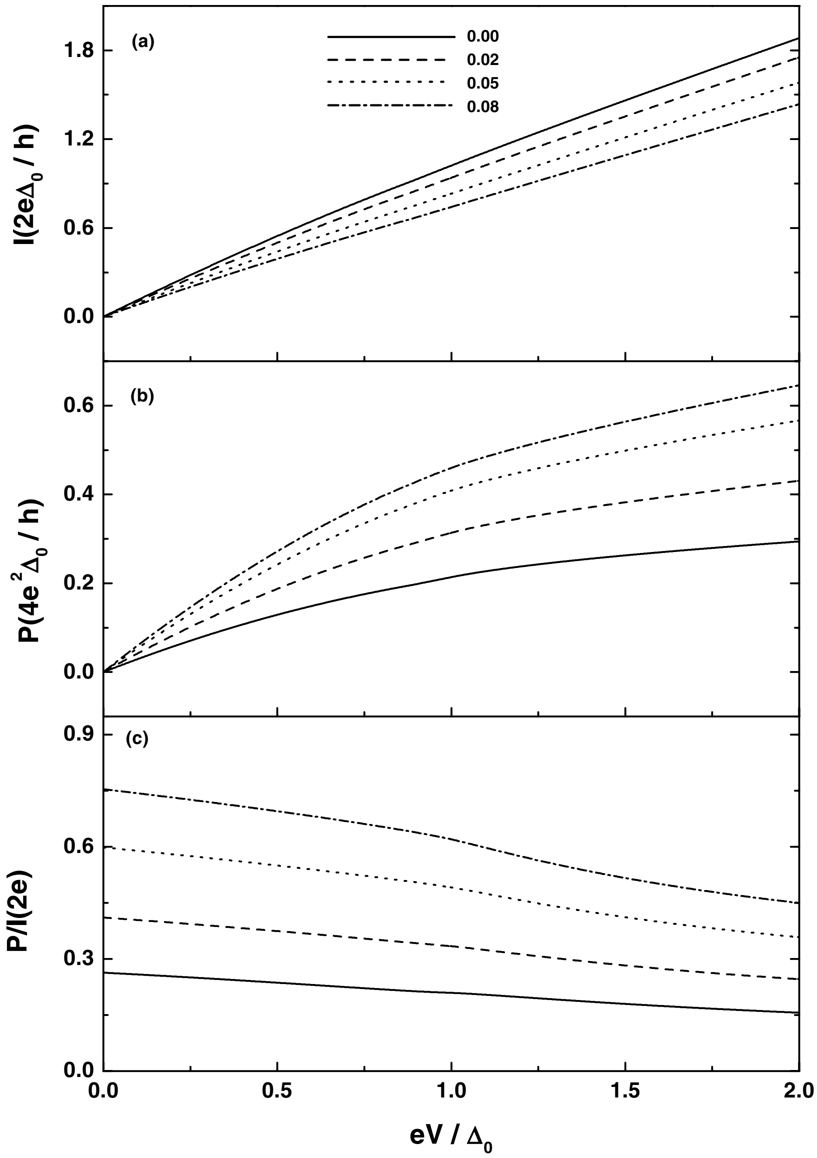

Figure 5. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of z_{20} in an FM/d-wave SC junction. Here $\alpha = \pi/4$, $z_{10} = 0$, and $h_0/E_F = 0.5$.

Finally, figure 6 and figure 7 show the barrier strength effects on the shot noise for the NM/s-wave SC and NM/d-wave SC junctions, respectively. For the high barrier ($z_{10} = 4$), the calculated results for the P/I ratio (dot–dashed curves in both figures) are in good agreement with the previous results for the NM/s-wave SC and NM/d-wave SC junctions [2–8]. For $eV < \Delta$, the P/I ratio in the d-wave case is quite different from that in the s-wave case; this outcome is attributed to the opening of noiseless midgap states in the d-wave SC. When the barrier height is lowered, both I and P increase, but their ratio P/I decreases.

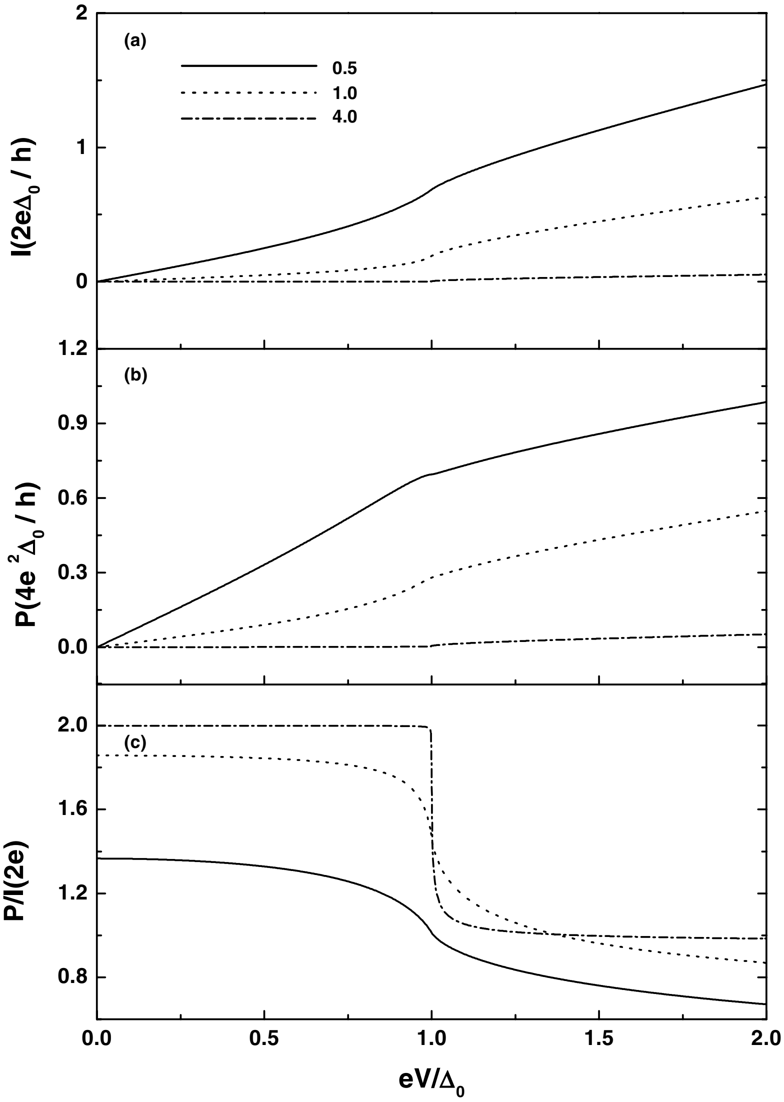

Figure 6. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of z_{10} in an FM/s-wave SC junction. Here $z_{20} = 0$ and $h_0/E_F = 0$.

4. Conclusions

In summary, we have investigated the effects of the interface roughness and exchange splitting on the shot noise in FM/d-wave SC and FM/s-wave SC junctions. It has been shown that the interface roughness leads to a decrease in the average current and an increase in the noise power-to-current ratio; while the exchange splitting in the FM leads to a decrease in the average current and shot noise power, and an increase in the noise power-to-current ratio at lower voltages. It is expected that the theoretical results obtained will be confirmed in future experiments. In the present model, we have neglected the spatial variation of the pair potential

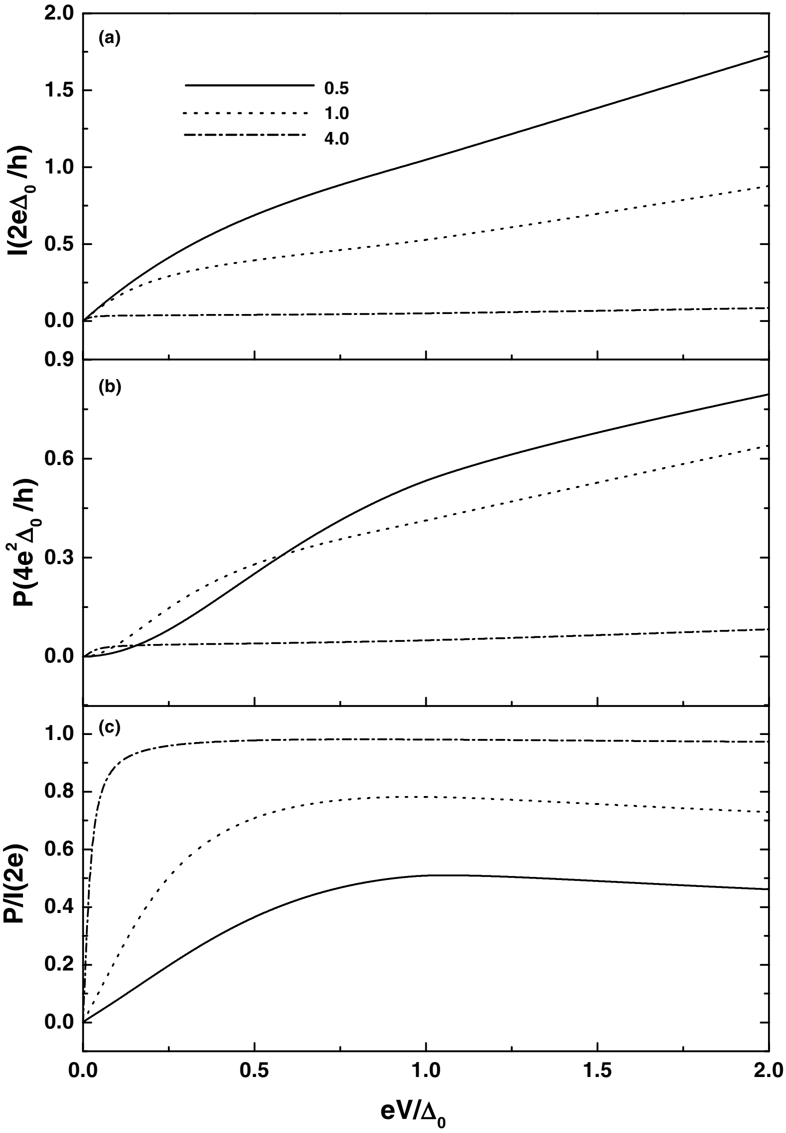

Figure 7. Average current I (a), shot noise power P (b), and the noise-to-current ratio P/I (c) as functions of bias with various values of z_{10} in an FM/d-wave SC junction. Here $\alpha = \pi/4$, $z_{20} = 0$, and $h_0/E_F = 0$.

in the SC due to proximity effects. Also, we have not considered the impurity scattering effect in the FM and the quasiparticle lifetime effect due to the inelastic scattering of the electrons. Inclusion of these effects would be necessary for a complete theory, which merits further study.

Acknowledgments

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